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Daniel Kenneth Villanueva
1838729

1. Find the center and the radius of convergence of:

$$\sum_{n=0}^{\infty} \frac{(4n)!}{2^n (n!)^4} (z + \pi i)^n$$

~~The center of convergence~~

The center of the series is ~~the~~ the series

$$z + \pi i = 0, \text{ which is:}$$

Center: $-\pi i$

The series could be expressed:

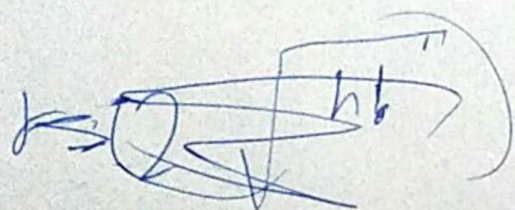
$$\sum_{n=0}^{\infty} C_n (z + \pi i)^n$$

Daniel Kennedy Villanova
 Also say to the Cauchy - A is derived
 Theorem the radius of convergence is!

$$r = \frac{1}{\limsup_{n \rightarrow \infty} \sqrt[n]{|c_n|}}$$

applying it to our series!

$$r = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{|c_n|}{|c_{n+1}|}}$$



$$r = \lim_{n \rightarrow \infty} \frac{(n!)^{\frac{4}{n}}}{(4n)!^{\frac{1}{n}}}$$

$$r = \lim_{n \rightarrow \infty} \frac{e^{\frac{4}{n} \ln n!}}{e^{\frac{1}{n} \ln (4n)!}} = \lim_{n \rightarrow \infty} \left(e^{\frac{4}{n} \ln n! - \frac{1}{n} \ln (4n)!} \right)$$

using Stirling's approximation

$$(n! = n \ln n - n + O(\ln n))!$$

of limits with n

$$r = \lim_{n \rightarrow \infty} 2 \left(e^{\frac{4}{n} (n \ln n - n + O(\ln n))} - \frac{1}{n} (4 \ln 4n - 4) \right)$$

$\lim_{n \rightarrow \infty} \frac{O(\ln n)}{n} = 0$

$$r = \lim_{n \rightarrow \infty} 2 \left(e^{4 \ln n - 4} - (4 \ln 4n - 4) \right)$$

$$r = \lim_{n \rightarrow \infty} 2 \left(e^{4 \ln n - 4} - 4 \ln 4n + 4 \right)$$

$$r = \lim_{n \rightarrow \infty} 2 \left(e^{-4 \ln 4} \right) = 2 \left(4^{-4} \right) = 2/25$$

There fore!

radius of convergence: 2^{-7}

David Lunnell Mathematics

2. Find the first four terms of the following Taylor Series!

$$e^{z(z-2)} \text{ centered at } z_0 = 1$$

From the Taylor series of e^x .

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$

Now substitute the degree of ~~center~~ ^{center} and the mapping $z \mapsto z(z-2)$

$$1 + (z(z-2) - 1) + \frac{(z(z-2) - 1)^2}{2} + \frac{(z(z-2) - 1)^3}{6}$$

$$z^2 - 2z - 1 = (z - i)^2$$

$$1 + (z - i)^2 + \frac{(z - i)^4}{2} + \frac{(z - i)^6}{6}$$

David Louis Villanueva
 3. Find the Power series for terms of the following
 meromorphic series!

$$\frac{z+2}{1-z^2}$$

$$\frac{z+2}{1-z^2} = \frac{z+1}{(1-z)(1+z)} + \frac{1}{1-z^2}$$

$$= \frac{1}{1-z} + \frac{1}{1-z^2}$$

from the ^{geometric} series formula!

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$\frac{z+2}{1-z^2} = \frac{1 + z + z^2 + z^3 + z^4 + z^5 + z^6 + \dots}{1 + z^2 + z^4 + z^6 + \dots}$$

$$= \frac{2 + z + 2z^2 + z^3 + 2z^4 + z^5 + 2z^6 + \dots}{1 + z^2 + z^4 + z^6 + \dots}$$

$$\frac{z+2}{1-z^2} = \boxed{2 + z + 2z^2 + z^3 + \dots}$$

g. And the Fourier series of $f(x)$, which is sorted to have the period 2π

$$f(x) = \begin{cases} x & \text{if } (-\pi < x < 0) \\ \pi - x & \text{if } (0 < x < \pi) \end{cases}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$G_0 = \frac{1}{2\pi} \left(\int_{-\pi}^0 \alpha dx + \int_0^{\pi} (\pi - \alpha) dx \right)$$

$$= \frac{1}{2\pi} \left(\frac{x^2}{2} \Big|_{-\pi}^0 + \pi x - \frac{x^2}{2} \Big|_0^{\pi} \right) = \frac{\pi^2}{2} - \frac{\pi^2}{2} = 0$$

$\epsilon_0 = 0$

Direkt Riemann willens

$$G_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{f(x)}{x} \cos nx \, dx$$

$$T_n = \frac{1}{\pi} \left(\int_{-\pi}^0 x \cos nx \, dx + \int_0^{\pi} (\pi - x) \cos nx \, dx \right)$$

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$$x \cos nx$$

$$1 \quad \frac{1}{n} \sin nx$$

$$0 - \frac{1}{n^2} \cos nx$$

$$G_n = \frac{1}{\pi} \left(\frac{1}{n} \sin nx - \frac{1}{n^2} \cos nx \right) \Big|_{-\pi}^0 + \frac{\pi}{n} \sin nx$$

$$G_{2k} = \frac{1}{\pi} \left(\frac{1}{n^2} + \frac{1}{n^2} \right) + \left(\frac{1}{n^2} - \frac{1}{n^2} \right) = 0$$

$$G_{2k-1} = \frac{1}{\pi} \left(-\frac{1}{n^2} - \frac{1}{n^2} \right) + \left(\frac{1}{n^2} - \frac{1}{n^2} \right)$$

$$= -\frac{2}{n^2 \pi} = -\frac{4}{(2k-1)^2 \pi}$$

$$\frac{1}{n} \sin nx + \frac{1}{n^2} \cos nx$$

Bestenfalls kann ich $\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin n\alpha \, d\alpha$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin n\alpha \, d\alpha$$

$$b_n = \frac{1}{\pi} \left(\int_{-\pi}^0 \alpha \sin n\alpha \, d\alpha + \int_0^{\pi} (\pi - \alpha) \sin n\alpha \, d\alpha \right)$$

0 I

$$\int \sin n\alpha \, d\alpha$$

$$= -\frac{1}{n} \cos n\alpha$$

$$= -\frac{1}{n^2} \sin n\alpha$$

$$b_n = \frac{1}{\pi} \left(-\frac{1}{n} \cos n\alpha - \frac{1}{n^2} \sin n\alpha \right) \Big|_{-\pi}^0 + \left(-\frac{1}{n} \cos n\alpha + \frac{1}{n^2} \sin n\alpha \right) \Big|_0^{\pi}$$

$$b_{2k} = \frac{1}{\pi} \left(-\frac{1}{n} + \frac{1}{n} \right) + \left(\frac{1-\pi}{n} - \frac{1-\pi}{n} \right) = 0$$

$$b_{2k-1} = \frac{1}{\pi} \left(-\frac{1}{n} - \frac{1}{n} \right) + \left(-\frac{1-\pi}{n} - \frac{1-\pi}{n} \right) = -\frac{4-2\pi}{n\pi}$$

$$b_{2k-1} = \frac{-4+2\pi}{(2k-1)\pi}$$

$$\frac{1}{\pi} \left(\frac{1-\pi}{n} - \frac{1-\pi}{n} \right) = 0$$

2.01 Lecture 11: Vectors

(9)

$$f(x) = \sum_{k=1}^{\infty} \left[\frac{-4 \cos((2k-1)x)}{(2k-1)^2 \pi} + \frac{(2\pi-4) \sin((2k-1)x)}{(2k-1) \pi} \right]$$

Therefore, the Fourier series for the function is:

$$\sum_{k=1}^{\infty} \left[\frac{-4 \cos((2k-1)x)}{(2k-1)^2 \pi} + \frac{(2\pi-4) \sin((2k-1)x)}{(2k-1) \pi} \right]$$